M12/5/MATHL/HP3/ENG/TZ0/DM

22127207

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

MATHEMATICS
HIGHER LEVEL
PAPER 3 – DISCRETE MATHEMATICS

Monday 7 May 2012 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the ***Mathematics HL and Further Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[60 marks]*.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 15]*

(a) Use the Euclidean algorithm to express $\gcd(123,\ 2347)$ in the form $123p + 2347q$, where $p,\ q \in \mathbb{Z}$. *[8 marks]*

(b) Find the least positive solution of $123x \equiv 1 \pmod{2347}$. *[3 marks]*

(c) Find the general solution of $123z \equiv 5 \pmod{2347}$. *[3 marks]*

(d) State the solution set of $123y \equiv 1 \pmod{2346}$. *[1 mark]*

2. *[Maximum mark: 7]*

The cost adjacency matrix for the weighted graph K is given below.

	A	B	C	D	E	F	G
A	0	5	2	0	0	0	0
B	5	0	0	0	7	0	0
C	2	0	0	4	4	0	0
D	0	0	4	0	2	0	9
E	0	7	4	2	0	4	3
F	0	0	0	0	4	0	1
G	0	0	0	9	3	1	0

Use Prim’s algorithm, starting at G, to draw two distinct minimal weight spanning trees for K.

3. *[Maximum mark: 8]*

The graph G has adjacency matrix $\boldsymbol{M}$ given below.

$$\begin{array}{c c} & \begin{array}{cccccc} \text{A} & \text{B} & \text{C} & \text{D} & \text{E} & \text{F} \end{array} \\ \begin{array}{c} \text{A} \\ \text{B} \\ \text{C} \\ \text{D} \\ \text{E} \\ \text{F} \end{array} & \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

(a) Draw the graph G. *[2 marks]*

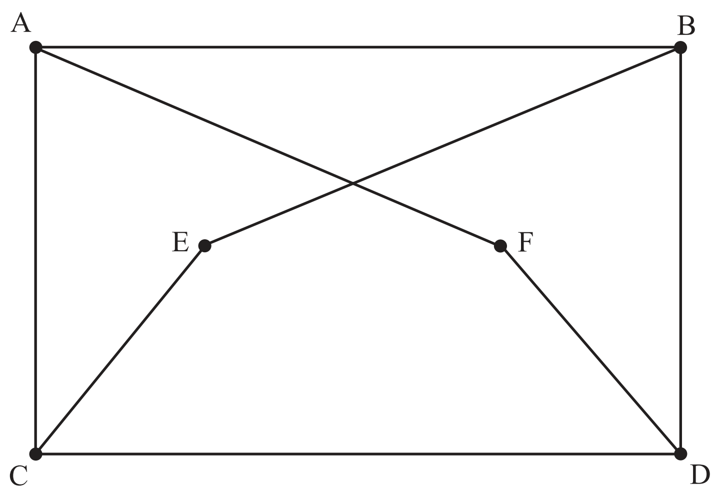
(b) What information about G is contained in the diagonal elements of $\boldsymbol{M}^2$? *[1 mark]*

(c) Find the number of walks of length 4 starting at A and ending at C. *[2 marks]*

(d) List the trails of length 4 starting at A and ending at C. *[3 marks]*

4. *[Maximum mark: 17]*

(a) Draw the complement of the following graph as a planar graph.



[3 marks]

(This question continues on the following page)

(Question 4 continued)

(b) A simple graph G has v vertices and e edges. The complement G' of G has e' edges.

(i) Prove that $e \le \frac{1}{2}v(v-1)$.

(ii) Find an expression for $e+e'$ in terms of v.

(iii) Given that G' is isomorphic to G, prove that v is of the form $4n$ or $4n+1$ for $n \in \mathbb{Z}^+$.

(iv) Prove that there is a unique simple graph with 4 vertices which is isomorphic to its complement.

(v) Prove that if $v \ge 11$, then G and G' cannot both be planar. *[14 marks]*

5. *[Maximum mark: 13]*

(a) Use the result $2003 = 6 \times 333 + 5$ and Fermat's little theorem to show that $2^{2003} \equiv 4 \pmod 7$. *[3 marks]*

(b) Find $2^{2003} \pmod{11}$ and $2^{2003} \pmod{13}$. *[3 marks]*

(c) Use the Chinese remainder theorem, or otherwise, to evaluate $2^{2003} \pmod{1001}$, noting that $1001 = 7 \times 11 \times 13$. *[7 marks]*
